# ABSTRACTS FOR MIDWEST PDE

Zachary Bradshaw:

*Existence problems for the Navier-Stokes equations*

In this talk we discuss the existence of solutions to the Navier-Stokes equations with non-decaying data in a local energy sense. The main focus is on discretely self-similar initial data, but more general results are also discussed. We will focus on the technical aspects of dealing with the nonlocal pressure term when the solution in view is non-decaying.

Siao-Hao Guo:

*Extension of two-dimensional mean curvature flow with free boundary*

In this talk we will consider a mean curvature flow of compact and embedded surfaces satisfying the Neumann boundary condition on a mean convex support surface. We will see that the flow can be extended so long as its mean curvature and perimeter stay uniformly bounded.

Vera Hur:

*Stokes waves in a constant vorticity flow: theory and computation*

Stokes in the 1800s made formal but far-reaching considerations about periodic waves at the surface of water, under the influence of gravity, propagating a long distance at a practically constant velocity without change of form. For instance, he observed that the crests become sharper and the troughs flatter, and that the so-called wave of greatest height, or the wave of extreme form, is distinguished by a 120 degree's peaking at the crest. The irrotational flow assumption is justified in many situations, and facilitates rigorous analysis and numerical computation. But rotational effects are significant in many others. I will review recent progress in the constant vorticity setting. Numerical findings include folds and gaps in the wave speed vs. amplitude plane, and a profile enclosing multiple bubbles of fluids.

Andrew Lorent:

*Null Lagrangian measures*

Compensated compactness is an important method used to solve nonlinear PDEs, in particular in the study of hyperbolic conservation laws. One of the simplest formulations of a compensated compactness problem is to ask for conditions on a compact set $\mathcal{K} \subset M^{m\times n}$ such that

$$\lim_{j\to\infty} \|\mathrm{dist}(Du_j, \mathcal{K})\|_{L^p} = 0 \text{ and } \sup_j \|u_j\|_{W^{1,p}} < \infty \ \Rightarrow \{Du_j\}_j \text{ is precompact in } L^p. \qquad (1)$$

Let $M_1, M_2, \ldots, M_q$ denote the set of all minors of $M^{m\times n}$. A sufficient condition for (1) is that any probability measure $\mu$ supported on $\mathcal{K}$ satisfying

$$\int M_k(X) d\mu(X) = M_k\left(\int X d\mu(X)\right) \text{ for all } k \qquad (2)$$

is a Dirac measure. We call measures that satisfy (2) *Null Lagrangian Measures* and following we denote the set of Null Lagrangian Measures supported on $\mathcal{K}$ by $\mathcal{M}^{pc}(\mathcal{K})$. For general $m, n$, a necessary and sufficient condition for triviality of $\mathcal{M}^{pc}(\mathcal{K})$ was an open question even in the case where $\mathcal{K}$ is a linear subspace of $M^{m\times n}$. We answer this question and provide a necessary and sufficient condition for any linear subspace $\mathcal{K} \subset M^{m\times n}$. The ideas also allow us to show that for any $d \in \{1, 2, 3\}$, $d$-dimensional subspaces $\mathcal{K} \subset M^{m\times n}$ support non-trivial Null Lagrangian Measures if and only if $\mathcal{K}$ has Rank-1 connections. This is known to be false for $d \geq 4$.

Null Lagrangian measures also arise in the study of certain nonlinear PDEs that are naturally associated to a set $K$ in the space of matrices, and triviality of $M^{pc}(K)$, often leads to nice properties of the PDEs. A particular examples comes from the method of compensated compactness, in which the Young measures can be viewed as Null Lagrangian measures. The ideas developed allow us to answer a question raised by Kirchheim, Müller and Sverák on the structure of $M^{pc}(K)$ for some nonlinear submanifold $K \subset \mathbb{R}^{3\times 2}$ that is associated to a well known $2 \times 2$ system of conservation laws with one entropy/entropy flux pair. This is joint work with Guanying Peng.

Yoichiro Mori:

*Immersed Elastic Filaments in Stokes Flow*

Problems in which immersed elastic structures interact with the surrounding fluid abound in biology, physics and engineering. Despite their scientific importance, analysis and numerical analysis of such problems are scarce or non-existent. In this talk, we consider the problem of an elastic filament immersed in a 2D or 3D Stokes fluid. We first discuss our recent results on the analysis of the immersed elastic interface problem in a 2D Stokes fluid (the Peskin problem). We prove well-posedness and immediate regularization of the elastic filament configuration, stability of steady states, criteria for global existence and discuss the implication of these results for numerical analysis. We will then discuss the immersed filament problem in a 3D Stokes fluid (the slender body problem). Here, it has not even been clear what the appropriate mathematical formulation of the problem should be. We propose a mathematical formulation for the slender body problem (the slender body PDE) and discuss well-posedness for the stationary version of this problem. Furthermore, we prove that the slender body approximation, introduced by Keller and Rubinow in the 1980's and used widely in the fluid-structure interaction community, provides an approximation to the slender body PDE with some error bound.

Toan Nguyen:

*Landau damping for screened Vlasov-Poisson on $\mathbb{R}^3$: a Lagrangian approach*

I shall present a joint work with D. Han-Kwan and F. Rousset, proposing a new Lagrangian proof of a recent result by Bedrossian, Masmoudi and Mouhot on Landau damping for screened Vlasov-Poisson systems on the whole space $\mathbb{R}^3$.

Dejan Slepcev:

*Proper regularizers for semi-supervised learning*

We consider a standard problem of semi-supervised learning: given a data set (considered as a point cloud in a euclidean space) with a small number of labeled points the task is to extrapolate the label values to the whole data set. In order to utilize the geometry of the dataset one creates a graph by connecting the nodes which are sufficiently close. Many standard approaches rely on minimizing graph-based functionals, which reward the agreement with the labels and the regularity of the estimator. Choosing a good regularization leads to questions about the relations between discrete functionals in random setting and continuum nonlocal and differential functionals. We will discuss how insights about this relation and results about the functionals provides ways to properly choose the functionals for semi-supervised learning and appropriately set the weights of the graph so that the information is propagated in a consistent way from the labeled points. This talk is based on joint works with Calder, Dunlop, Stuart and Thorpe.

Milena Stanislavova:

*Asymptotic behavior of semigroups generated by Hamiltonian linearizations*

In this talk, we showcase the advantages of using uniform resolvent estimates close to the imaginary axis to investigate the asymptotic behavior of the solution semigroups in the case of spectrally stable generators. First, we consider the spectrally stable periodic steady states of the Lugiato-Lefever model of optical fibers and show that the solution semigroup obeys optimal exponential decay estimates. Moreover, these resolvent estimates can be used to prove that the solutions are asymptotically stable with phase. Our second example is motivated by the linearization about standing waves of the NLS on a periodic domain. In this situation, we prove that the semigroup grows at most polynomially in time and is uniformly bounded on $L^2$ when all eigenvalues of the generator have the same algebraic and geometric multiplicities.

Ian Tobasco:

*The cost of crushing: curvature-driven wrinkling of thin elastic shells*

How much energy does it cost to stamp a thin elastic shell flat? Motivated by recent experiments on the wrinkling patterns formed by thin shells floating on a water bath, we develop a rigorous method via Gamma-convergence for evaluating the cost of crushing to leading order in the shell's thickness and other small parameters. The experimentally observed patterns involve regions of well-defined wrinkling alongside totally disordered regions where no single direction of wrinkling is preferred. Our goal is to explain the appearance and lack thereof of such wrinkling domains. The basic objects that emerge in the limit are short maps from the mid-shell into the plane, and defect measures which encode the wrinkling patterns. Convex analysis of the limiting energy yields a new boundary value-like problem whose solutions completely characterize optimal wrinkling patterns via their defect measures. Optimal wrinkling patterns are in general non-unique. Nevertheless, in some cases their restrictions to certain sub-domains are uniquely determined, and explicit formulas exist.

Monica Torres:

*Divergence-measure fields: Gauss-Green formulas and normal traces.*

The Gauss-Green formula is a fundamental tool in analysis. In this talk we present new Gauss-Green formulas for divergence-measure fields (i.e.; vector fields in $L^p$, $1 \leq p \leq \infty$ whose divergence is a Radon measure) which hold on rough open sets of finite perimeter or general open sets. This is a joint work with Gui-Qiang Chen (University of Oxford), Giovanni Comi (Scuola Normale Superiore, Pisa) and Qinfeng Li (University of Texas, San Antonio).

Sam Walsh:

*Capillary-gravity water waves with exponentially localized vorticity*

In this talk, we discuss recent success in establishing the existence of solutions to the water wave problem with exponentially decaying vorticity. These are two-dimensional stationary waves in a finite-depth body of water beneath vacuum. An external gravitational force acts in the bulk, and the effects of surface tension are felt on the air-sea interface. Our approach involves modeling the corresponding stream function as a spike solution to a singularly perturbed elliptic PDE. This is joint work with Mats Ehrnstrom (NTNU) and Chongchun Zeng (Georgia Tech).